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Proportion as the Timeless Essence of Mughal Architecture

*La proporción como esencia atemporal
de la arquitectura mogol*

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Keywords | Palabras clave | Palavras chave

Mughal design methods, Tomb of Akbarabadi Mahal, Tomb of Fatehpuri Begum, Taj Mahal, Badshahi Mosque

Métodos de diseño mogoles, Tumba de Akbarabadi Mahal, Tumba de Fatehpuri Begum, Taj Mahal, Mezquita Badshahi

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Abstract | Resumen | Resumo

This paper presents a geometric analysis of four seventeenth-century Mughal buildings in an attempt to decipher the design methods used by Mughal architects to create their masterpieces. The geometric analysis is placed in the framework of the *Hasht Bihisht* (Eight Paradises) plan-form, the archetypal ground-plan of Mughal architecture. Categorized as a type of “nine-square mandala” diagram, the *Hasht Bihisht* is a symbol of the cosmos, i.e., a harmoniously created universe. Our findings are a continuation of those of scholars of Central Asian Timurid architecture and analogous to what is common to traditional architecture generally, down to our times, as will be shown through the work of a hereditary master mason. In conclusion, it is proposed that these design methods be interpreted in the light of traditional cosmology.

Este artículo presenta un análisis geométrico de cuatro edificios mogoles del siglo XVII con el objetivo de descifrar los métodos de diseño empleados por los arquitectos mogoles para crear sus obras maestras. El análisis geométrico se sitúa en el marco del esquema *Hasht Bihisht* (“Ocho Paraísos”), la planta arquetípica de la arquitectura mogol. Clasificado como un tipo de diagrama de “mandala de nueve cuadrados”, el *Hasht Bihisht* simboliza el cosmos, es decir, un universo creado en armonía. Nuestros hallazgos dan continuidad a las investigaciones sobre la arquitectura timúrida de Asia Central y se corresponden con los principios comunes a la arquitectura tradicional en general, vigentes hasta nuestros días, como se mostrará a través del trabajo de un maestro albañil heredero de esta tradición. En conclusión, se propone interpretar estos métodos de diseño a la luz de la cosmología tradicional.

Este artigo apresenta uma análise geométrica de quatro edifícios mogóis do século XVII, com o objetivo de decifrar os métodos de concepção utilizados pelos arquitetos mogóis na criação de suas obras-primas. A análise geométrica é enquadrada no esquema *Hasht Bihisht* (“Oito Paraísos”), a planta arquetípica da arquitetura mogol. Classificado como um tipo de diagrama de “mandala de nove quadrados”, o *Hasht Bihisht* simboliza o cosmos, ou seja, um universo criado em harmonia. Nossas conclusões dão continuidade às pesquisas sobre a arquitetura timúrida da Ásia Central e são análogas aos princípios comuns da arquitetura tradicional em geral, vigentes até os nossos dias, como será demonstrado através do trabalho de um mestre pedreiro hereditário. Em conclusão, propõe-se que esses métodos de concepção sejam interpretados à luz da cosmologia tradicional.

Introduction

“When one sees eternity in things that pass away, then one has pure knowledge”

Bhagvad Gita

“The good, of course, is always beautiful, and the beautiful never lacks proportion”

Plato

“The transcendent world is the world of loveliness and beauty, and the source of loveliness and beauty is harmony/proportion (tanashub)...”

Imam Ghazālī

Proportion has been seen as the essence of beauty across cultures, as evidenced in the above quotations, while beauty itself is seen as an aspect of the “divine” and hence as a reflection of the “timeless.”

“Its boundaries [lit. sides] are Eternity without beginning or end,
While its stairs are Time’s total aggregate of years.

Since its lustrous stone is entirely free from blemish,
The edifice appears as unified as one perfect bubble.”

Description of the Taj Mahal by court poet Kalim in the *Padshah Namah* (Begley, Wayne, and Desai 1984)

This study has been undertaken by a practicing architect attempting to develop ways of employing these methods in contemporary practice, based on a knowledge of their traditional use and their artistic function—to create an architecture which reflects something of the timeless quality of art.



Figure 1: Babur supervising the laying out of the Garden of Fidelity, painting ca. 1590 (Victoria and Albert Museum)

The Mughals & Mughal Architecture

Founded in 1526 by Babur (1483-1530), the Mughals were a dynasty of Timurid¹ descent whose presence in India lasted from 1526 to 1857. It was Akbar who in the second half of the sixteenth century (1556-1605) laid the administrative and cultural foundations which defined the subsequent Mughal rule in India. Shah Jahan's reign in 1628-1658 saw the Mughal empire reach its heyday. Its largest expansion came under Aurangzeb (1658-1707), the last of the great Mughal emperors, after whose death the dynasty declined until 1757, when it disintegrated into princely states only nominally subservient to the king in Delhi. The last of these kings, Bahadur Shah Zafar, was deposed by the British in 1857.



Figure 2: Three Timurid madrasahs, Registan Square, Samarkand, Uzbekistan, Ulugh Beg Madrasah (1417-1420), Sher-Dor Madrasah (1619-1636), and Tilya-Kori Madrasah (1646-1660) (Geoff Henson)



Figure 3: I'timād-ud-Daulah, Agra (Muhammad Mahdi Karim)

The Mughal court attracted scholars, painters, architects, poets, calligraphers, and statesmen from all over Iran, Central Asia, Transoxiana, and the Subcontinent. The royal family and court not only patronized this galaxy of artists and intellectuals but took a personal interest in many arts and sciences.

It is thus no wonder that in this rich artistic and intellectual environment coupled with enormous wealth, Mughal architecture created a supremely confident style synthesizing Transoxianian, Timurid, Indian, Persian, and even some European elements (Koch, Ebba 1991: 13).

The defining influence on their architecture, though, was Timurid. To quote the Austrian scholar Professor Ebba Koch (1991: 13):

Since the Mughals were direct heirs to the Timurids, the sustaining element of their architecture, especially during the initial phase, was Timurid (in the older literature often considered to be "Persian"). [...] A fact that is not generally recognised is that essential ideas of Timurid architecture, such as the perfect symmetry of plan reflected consistently in the elevations, as well as complex vault patterns, came to fruition much more in Mughal architecture than in Safavid Iran, which was also heir to the same tradition.

Framework: *Hasht Bihisht* (Eight Paradises) Plan-form, Principles and Design Methods

According to Koch, the archetypal Mughal ground-plan is the *Hasht Bihisht* plan-form of Persianate Islamic architecture (Koch 1991: 45). Found in other traditions, it is sometimes named the "nine-square mandala" or "cosmogram"—the *Vastu-Purusha* mandala of Indian temple architecture or the *Ming-Tang* square of traditional Chinese design. Through these various names in different traditions and the related mythologies, this ground-plan is simultaneously seen as a symbol of paradise and of the cosmos in its qualitative aspect. As the cosmos is regarded as a reflection of a Divine Principle, this architecture ultimately reflects the qualities of that Principle with its cosmic symbolism, through a conception common to all premodern and sacred traditions, in which the cosmos is a manifestation of the Creator.

"I was a Hidden Treasure and wished to be known, so I created the creation that I might be known."

Hadith Qudsi—God's words as reported by the Prophet of Islam

"Crazy Horse dreamed and went into the world where there is nothing but the spirits of all things. That is the real

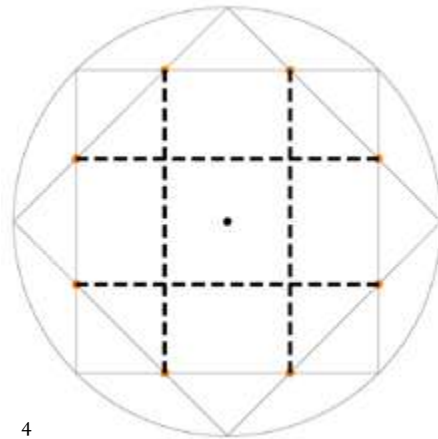


Figure 4: *Hasht Bihisht* – Geometry

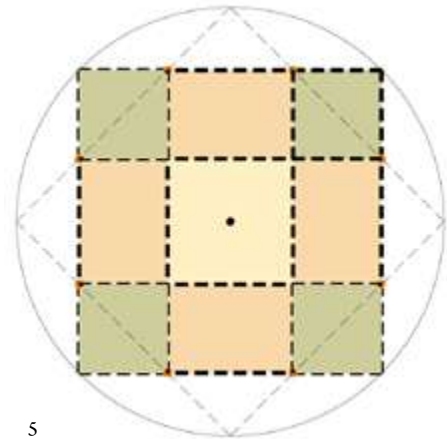


Figure 5: *Hasht Bihisht* – Spatial configuration

world that is behind this one, and everything we see here is something like a shadow from that world.”

Black Elk Speaks (Neihardt 1972)

“It all seems ‘real’, but as it is constantly changing, it is not Real. Due to this *Maya* mentality (the illusion that the world is Real), people do not look beyond the veil of illusion to Me, the unchanging consciousness, the Absolute Reality ... the very basis of it all.”

Bhagavad-Gita

The archetypical form of this plan is a central square surrounded on the four major axes by four rectangles and at the corners by smaller squares. Geometrically, this is derived from the intersection points of two squares at 45 degrees to each other, which give the vertices of a regular octagon (Figs. 4 and 5). The typical divisions of elevation and section are also projections of this division from the ground (Fig. 6). This archetypical arrangement is employed at all scales from facades as a whole to facade elements and its consistent application generates a sense of unity and wholeness typical of traditional architecture.

The *Hasht Bihisht* plan is especially prominent in Mughal architecture, which adopted and developed this model (Koch 1991: 27). It is within the *Hasht Bihisht* “spatial framework” that “elements” such as arches, domes, *jharokas* (bay windows), and geometric patterns are placed. Also constitutive of Mughal architecture are aesthetic “principles of design”: balance, symmetry, harmony, emphasis on a central element (achieved through odd-numbered divisions of elevations and within elements). All these principles again reflect the conception of beauty outlined above and in our opening quotations. Finally are the practical “methods” or tools of design—including geometric proportioning—used to give physical form to these principles².

Geometric Analyses

Geometric analyses of two small and relatively simple Mughal buildings are presented below to highlight the basic design approach and methods used. The first is of the Eastern Subsidiary Tomb of Akbarabadi Mahal (d. 1677) or “Saheli Burj”, in the Taj complex (Fig. 7).

Typical Building Typology:

The ‘Nine Square’
Hasht Bihisht
Plan-type

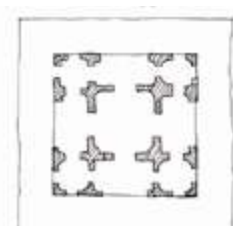
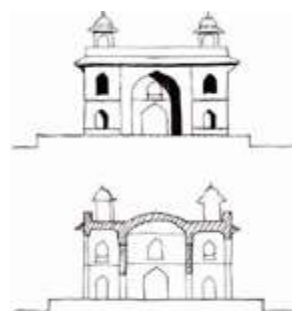


Figure 6: *Hasht Bihisht* in Mughal architecture – Schematic example of a typical Mughal pavilion

Main features:



- Double storied corners
- Double-height iwans on Cardinal Axes

- Double-height central space

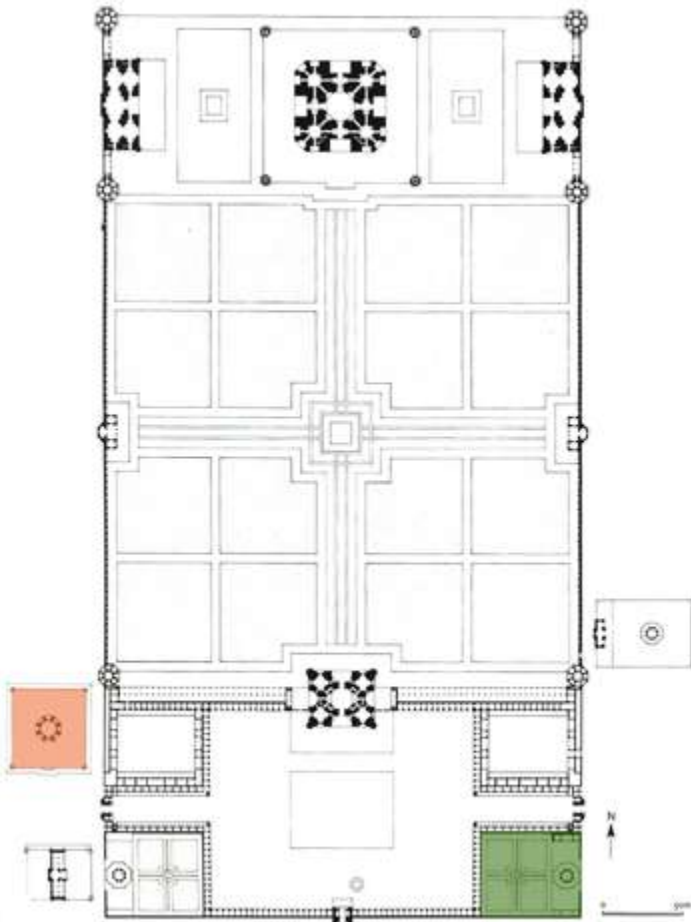


Figure 7: Taj Mahal complex with Saheli Burj highlighted in green and the tomb of Fatehpuri Begum in orange (Koch 2006: 114)

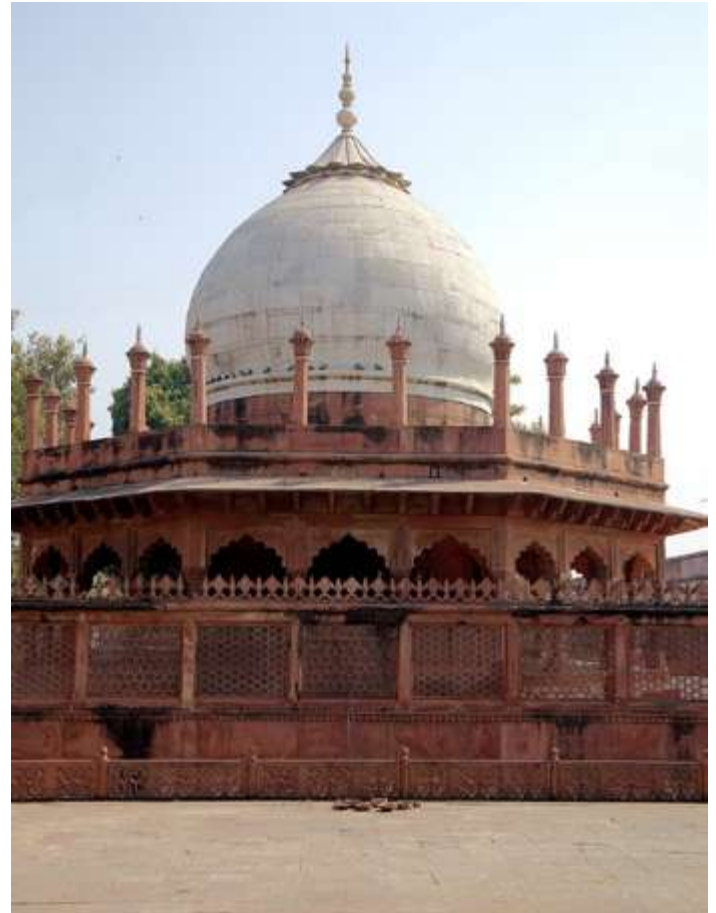


Figure 8: Eastern Subsidiary Tomb of Akbarabadi Mahal (Saheli Burj) (American Institute of Indian Studies)

According to Koch, “the two enclosures that flank the *Jilaukhana* (forecourt) on the south are funerary complexes held to be those of lesser wives of Shah Jahan” (Koch 2006: 120). The Eastern Subsidiary Tomb could well be that of Akbarabadi Mahal Begum, who died in 1677 (Koch 2006: 121).

Saheli Burj: Geometric Analysis

Analysis of the Plan on a Drawing by Ebba Koch (2006: 121)

Step 01. A series of diminishing inscribed rotating squares gives us a series of regular octagons from the veranda perimeter to the tomb chamber perimeter (Fig. 9).

Step 02. Adding the next two inscribed rotating squares gives us the tomb chamber’s inner perimeter as well as the wall thickness (Fig. 10).

Step 03. Continuing the series of inscribed rotating squares inward gives us the grave length as 2.66 gaz^3 , and the intersections of the last two rotating squares gives the grave

width as 1 gaz . The length of the grave platform, 3.66 gaz , is given by the diagonal of the last rotated (red) square (Fig. 11).

Step 04. Extending the lines of these last rotating squares outward marks out the width of the recessed arched panels (pseudo-*pishtaqs*, in Koch’s view) on the tomb chamber’s outer elevation (Fig. 12).

Step 05. Extending the lines of the second-last series of rotating squares outward marks out the width of the tomb chamber entrances (Fig. 13).

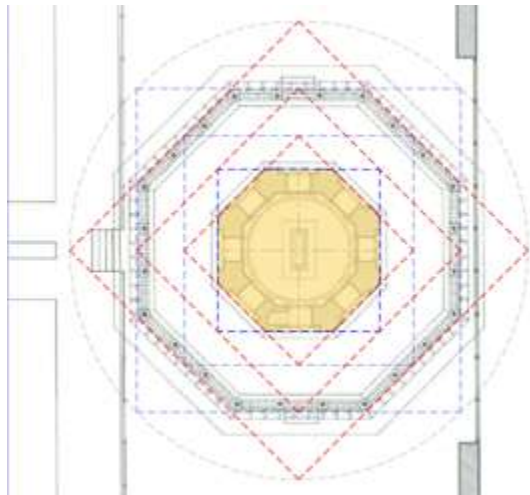


Figure 9: Geometric analysis of Saheli Burj – Plan – Step 01

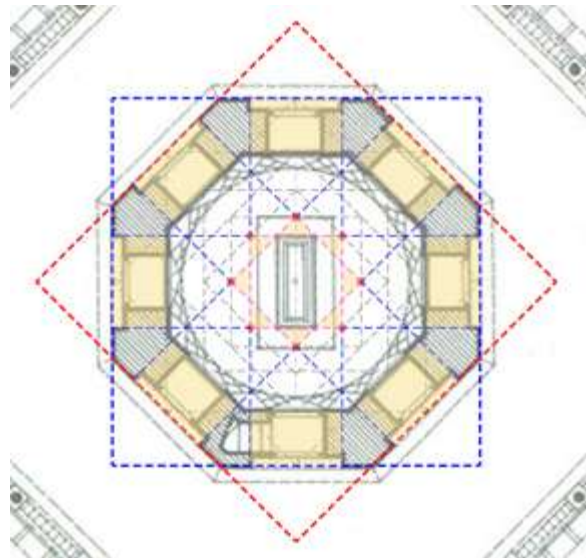


Figure 12: Geometric analysis of Saheli Burj – Plan – Step 04

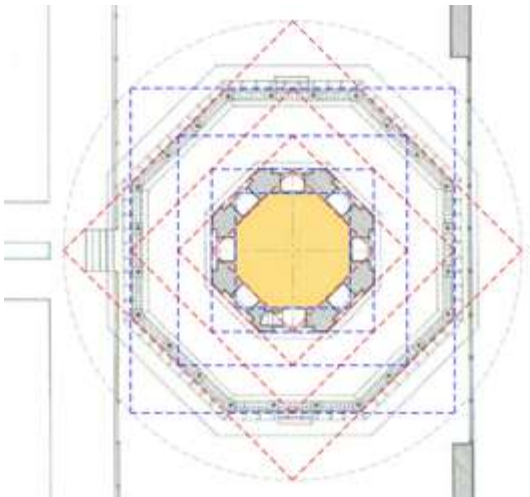


Figure 10: Geometric analysis of Saheli Burj – Plan – Step 02

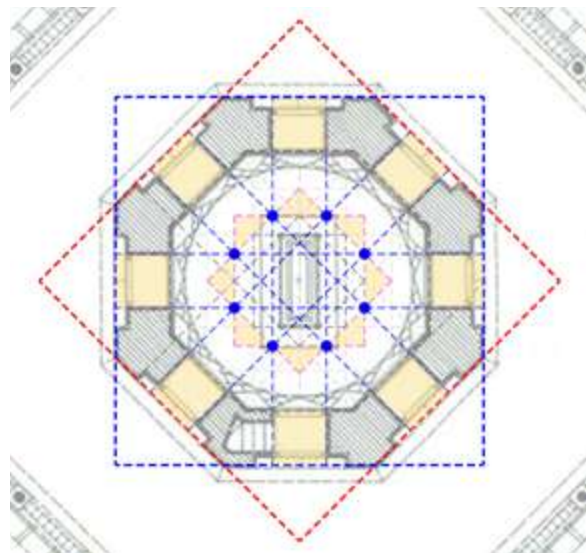


Figure 13: Geometric analysis of Saheli Burj – Plan – Step 05

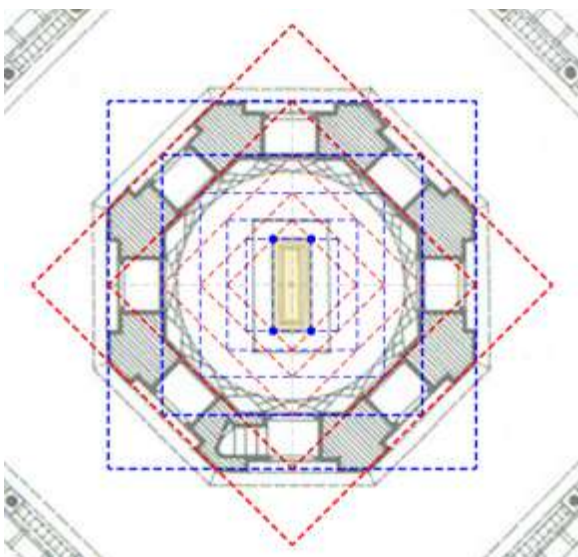


Figure 11: Geometric analysis of Saheli Burj – Plan – Step 03

Analysis of the Section on a Drawing by Ebba Koch (2006: 121)

Step 01. This time, starting with two rotating squares from the inner chamber (highlighted in magenta), a series of circumscribing rotating squares expanding outward give the main proportions of the section (Fig. 14).

Step 02. The height of the room is equal to its width (7 gaz), the limits of the dome and its *maujpa* (the inverted lotus element below the finial, *kalash*), and some minor heights: roof, parapet, parapet molding, and the lotus tips on the thin minarets at the eight corners of the veranda (Fig. 15).

Step 03. Duplicating the ground-floor series of circumscribing rotated squares on the first floor, starting

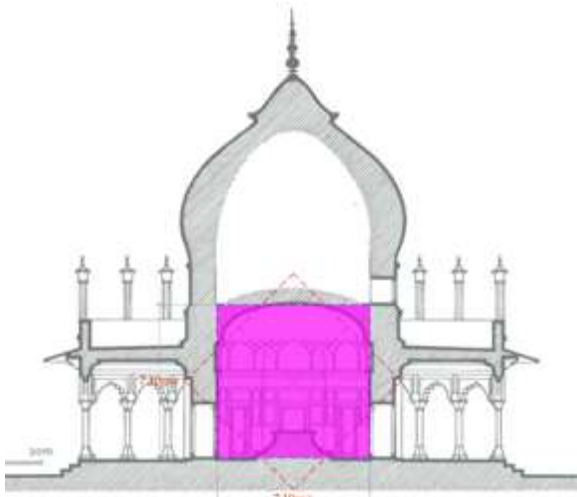


Figure 14: Geometric analysis of Saheli Burj – Section – Step 01

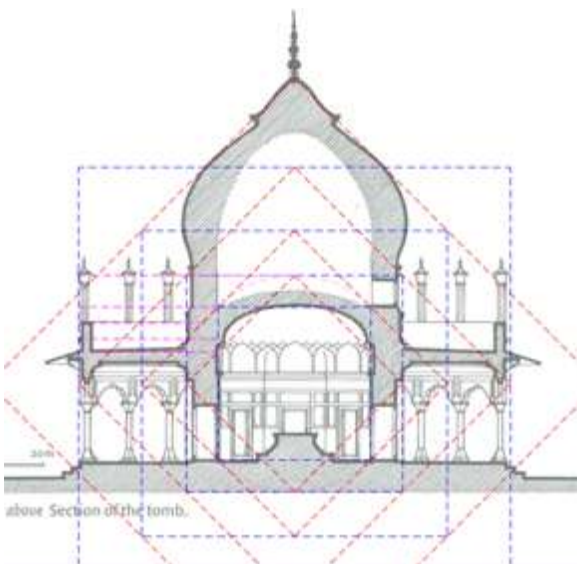


Figure 15: Geometric analysis of Saheli Burj – Section – Step 02

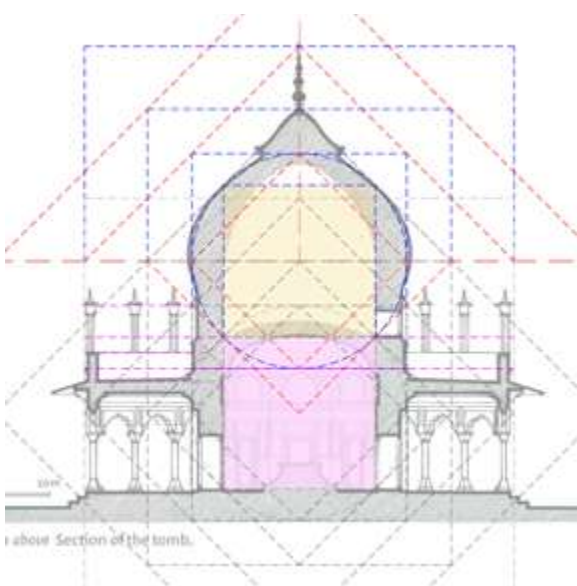


Figure 16: Geometric analysis of Saheli Burj – Section – Step 03

with the square highlighted in yellow, gives the heights of the finial and inverted lotus elements. The intersection of the two systems gives the width of the lotus elements (Fig. 16).

Gaz-Grid Analysis (According to the Shahjahani Gaz) on a Drawing by Ebba Koch (2006: 121)

Plan (Fig. 17):

Grave width	= 1 gaz
Grave platform width	= 2 gaz
Chamber inner wall	= 3 gaz
Chamber width	= 7 gaz
Chamber outer wall	= 4.25 gaz
Chamber outer width	= 10.28 gaz
Chamber outer width including overhang	= 11 gaz
Veranda ambulatory	= 5 gaz
Veranda inner width	= 19 gaz
Veranda outer width	= 20 gaz
Building width with overhang	= 23 gaz

Section (Fig. 18):

Heights:

Inner cornice	= 4 gaz
Ceiling	= 7 gaz
Dome drum	= 2 gaz
Drum base	= 2 gaz
Pencil minarets	= 4 gaz
Roof to lotus finial	= 11 gaz
Lotus finial	= 2 gaz
Top finial	= 3 gaz
Dome	= 7 gaz
Ground to parapet top	= 7 gaz
Floor to finial top	= 21 gaz

Widths:

Veranda outer width	= 20 gaz
Lotus finial	= 4 gaz
Dome pedestal	= 10 gaz
Total width	= 23 gaz
Grave width	= 1 gaz
Grave platform width	= 2 gaz
Chamber inner wall	= 3 gaz
Chamber width	= 7 gaz
Chamber outer wall	= 4.25 gaz
Chamber outer width	= 10.28 gaz
Chamber outer width including overhang	= 11 gaz
Veranda ambulatory	= 5 gaz
Veranda inner width	= 19 gaz
Veranda outer width	= 20 gaz
Building width with overhang	= 23 gaz

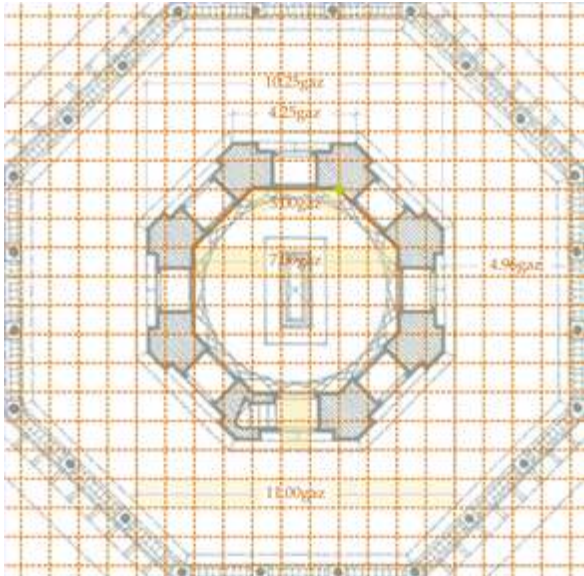


Figure 17: Geometric analysis of Saheli Burj – Plan – Grid analysis

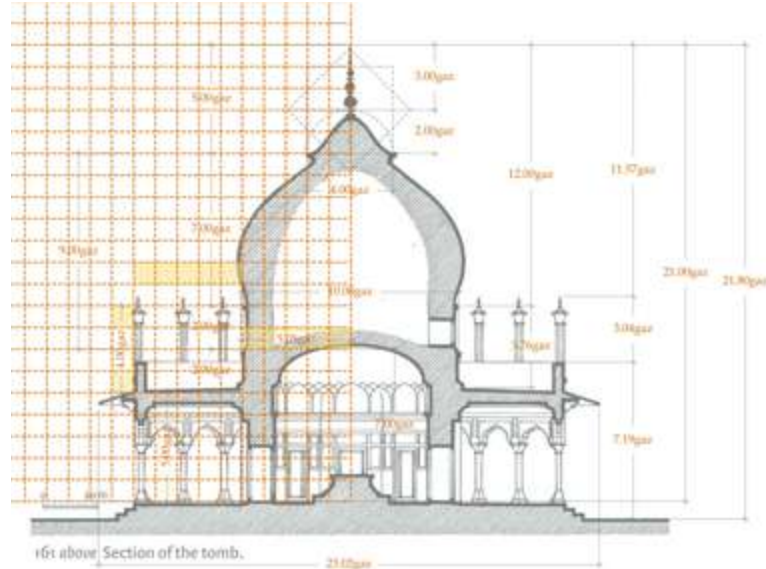


Figure 18: Geometric analysis of Saheli Burj – Section – Grid analysis

Geometric Analysis of the Tomb of Fatehpuri Begum or Satti-un-Nisa Khanum

Our second analysis is of the plan of the tomb of Fatehpuri Begum or Satti-un-Nisa Khanum (d. 1647) (Fig. 19), by the west wall of the Taj Mahal complex (Fig. 7, highlighted

in orange). It has also been identified as that of Satti-un-Nissa Khanum, lady-in-waiting of Mumtaz Mahal (Koch 2006: 120).

Analysis of the Plan on a Drawing by Ebba Koch

Starting from the tomb chamber, one set of rotating squares gives us the wall thickness, but the next set of circumscribing (expanding) squares misses the building's outer limit by a small margin. As we will see in the gaz-grid analysis below, this is to adjust to the underlying linear-yard (gaz) grid. Inscribing (inward) we get a set of rotating squares, and extending their intersection points gives us a square in which the length of the grave sits exactly, while the rotation of this inner square and the mutual intersections give us the grave width (Fig. 20).

Gaz-Grid Analysis (According to the *Shahjahani Gaz*) on a Drawing by Ebba Koch

Grave width	= 1 gaz
Grave length	= 2.5 gaz
Chamber inner entrance	= 1.5 gaz
Chamber width	= 9 gaz
Chamber wall length	= 4 gaz
Interior recessed arched panels width	= 3 gaz
Tomb entrance width	= 4 gaz
Tomb entrance depth	= 2.25 gaz
Tomb outer side	= 7 gaz
Building width	= 17 gaz

Note: the blue dots and dashed lines mark grid intersections and their connecting lines (Fig. 21).

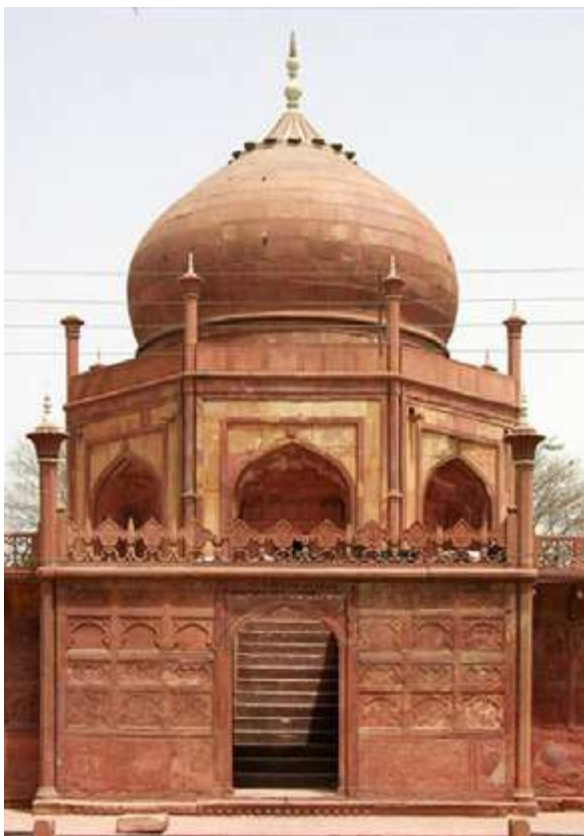


Figure 19: Tomb of Fatehpuri Begum, Agra (ArchNet)

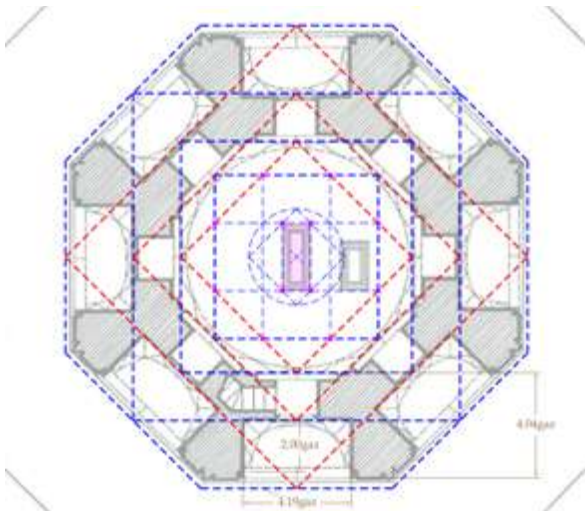


Figure 20: Geometric analysis of the tomb of Fatehpuri Begum, Agra - Plan

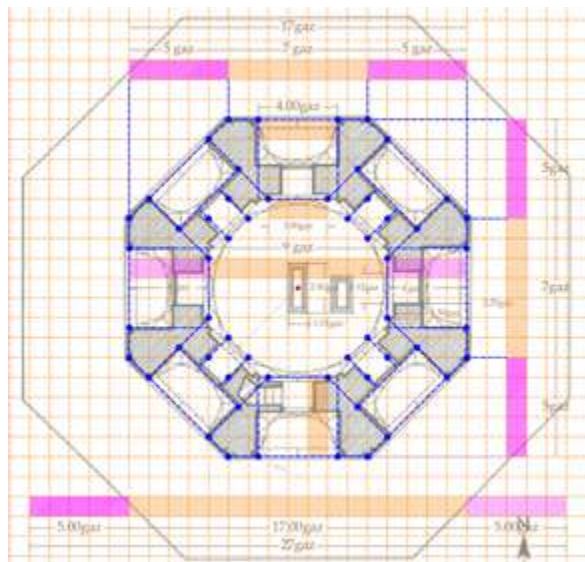


Figure 21: Gaz-grid analysis (left), according to the shahjahani gaz of the tomb of Fatehpuri Begum, Agra - Plan

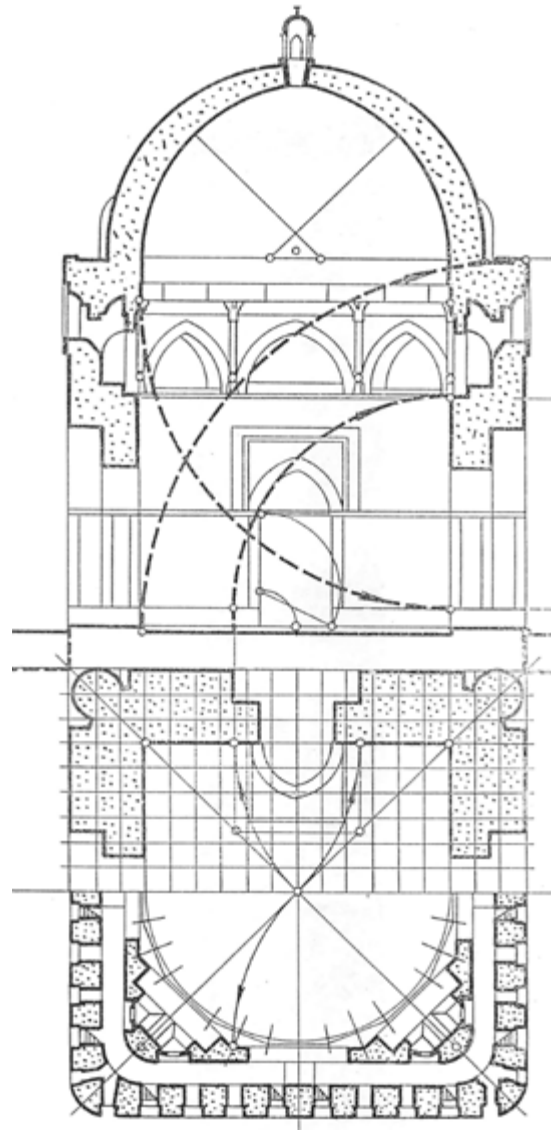


Figure 22: Tomb of Ismail Samani, Bukhara, 907 AD. Section and plan (Bulatov 1988)

Interpretation in the Light of Bulatov’s Study

Thus, a geometric system based on inscribed squares and a modular system based on the Mughal yard (the *ilahi gaz*⁴) are simultaneously present in these buildings. This relates directly to the design process and geometric systems presented in the study of Timurid architecture by Lisa Golembeck and Donald Wilber, *The Timurid Architecture of Iran and Turan* (1988). Their comments in turn draw on the work of the Uzbek scholar Mitxat Sagadatdinovich Bulatov, *Geometric Harmonisation in the Architecture of Central Asia, IX – XV Centuries* (1978).

According to Golombek and Wilber (1988), the design of a traditional Timurid building would use two processes simultaneously, namely “geometric” and “analytical.”

Thus, first the design was drawn more or less theoretically, according to geometric proportions, and then the analytical process was applied, and one dimension was selected as a module. In Timurid buildings the module was equal to the wall thickness, also equal to or commensurate with the *gaz*, which in early Timurid buildings averaged around 24 inches (61 cm). The module was further subdivided for details. The architect could therefore specify measurements in terms of rational numbers (Golembeck and Wilber 1988: 139).

In the system described by Bulatov, firstly a measurement—the “generative unit”—had to be chosen. This was a single element, often the most prominent feature (e.g., the central chamber of the tombs analyzed above). All important dimensions in plan, elevation, and section would be

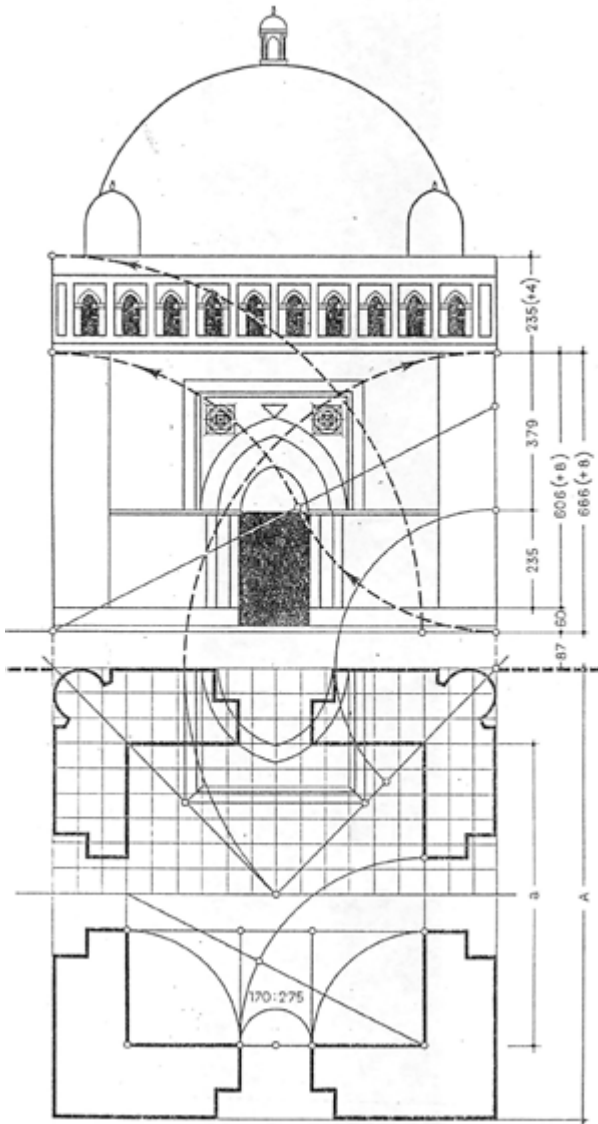


Figure 23: Tomb of Ismail Samani, Bukhara, 907 AD. Elevation and plan (Bulatov 1988)

commensurable with this. That two systems, analytical and geometric, were used is borne out by analyses of buildings exhibiting both proportional and modular systems. Figures 22 and 23, taken from Bulatov’s study, illustrate the use of such systems in the tomb of Ismail Samani in Bukhara (907 AD).

Historically, this “modular” aspect of architecture can be seen at work from some of the earliest architectural drawings available to us (Fig. 10):

This fragmentary architectural drawing shows part of a small shrine [...]. The measurements of the walls are written from right to left, and each begins with a hieroglyph in the shape of a forearm, which means “cubit,” the Egyptian unit of length. This symbol is followed, in



Figure 24: Architectural Drawing of a Garden. Ancient Egypt, ca. 1550–1295 BC (The Metropolitan Museum of Art)



Figure 25: Plan or drawing tablet excavated in Sippar-Yahrurum (Tell Abu Habbah), dated to the Neo-Babylonian period (ca. 626-539 BC) and now in the British Museum, London

each case, by a number: an elongated semicircle stands for ten, and a stroke stands for one. Thus, the width of the orchard is thirty-two cubits (The Metropolitan Museum of Art).

Similarly, the use of the grid as a tool for this modular aspect is attested to in one of the earliest architectural drawings (Fig. 25). This shows a gridded plan on a clay tablet from the Neo-Babylonian period (ca. 626-539 BC), while the ancient Indian architectural manuals known as the *Vastu Shastras* also attest to the use of grids and modular proportioning (Meister 1985). Analyses of classical Greek architecture have likewise shown the use of both these methods, i.e., “geometric” and “modular” (Jones 2006; Leonardis 2016).

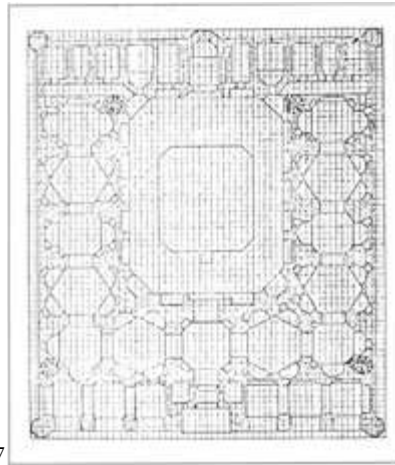
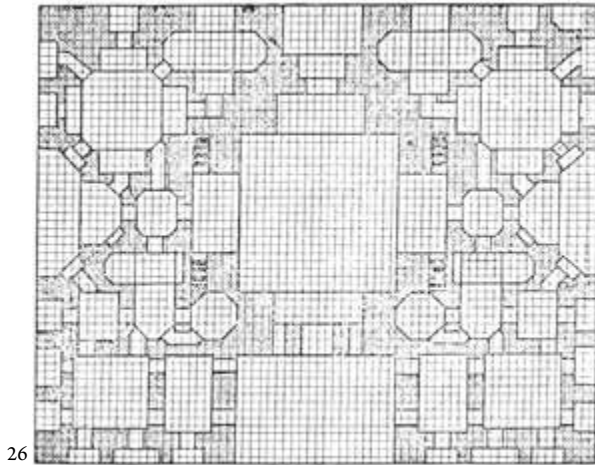


Figure 26: Drawing by the Bukharan Master, probably sixteenth century (Holod 1988)

Figure 27: Drawing by the Bukharan Master, probably sixteenth century (Holod 1988)

In the case of Mughal architecture, we have the famous Mughal miniature (Fig. 1) showing the first Mughal king, Babur, supervising the laying out of a garden in Kabul while an architect stands by with what appears to be a drawing board with red coating on which a grid can be seen engraved (probably with a layout drawing), pointing to this same tradition. The use of grids by Timurid architects is also attested to, for example by the drawings of the “Bukharan Master” (Figs. 26 and 27).

What is evident from Bulatov’s studies and also our own analyses above is that the grid’s primary function is to convert a geometrically generated design into whole numbers and fractions commensurable with the yard (the Mughal *ilahi gaz*) and thus easily usable on site, as opposed to the irrational numbers which a purely geometrically generated proportioning system would yield.

The use of graph paper is most significant. It tells us that the draftsmen worked with compasses and straight edges, the ancient tools of Euclidean geometry. Any geometric form was quickly plotted out, making transformations of inscribed polygons, diagonals, and segments. Bulatov sees this series of transformations as all-important, extending into three dimensions and determining elevations (Holod 1988: 6).

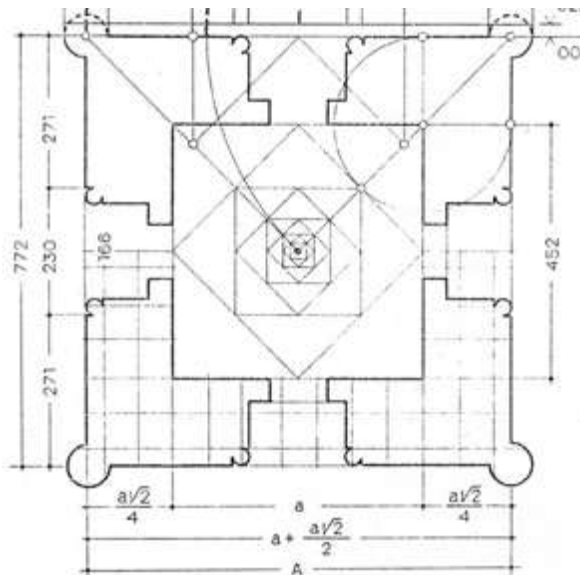


Figure 28: Mausoleum of Aisha-Bibi, part of Bulatov’s illustration for Table 1 (Bulatov 1988: 111)

See also Yasser Tabba’s two case studies (1988) from Aleppo and Damascus on the use of the modular grid as well as of geometric proportioning. Necipoğlu (1996) covered the same ground as Holod (1988) but with the addition of Ottoman plans using modular grids. Koch’s work over more than two decades of measuring and drawing Mughal buildings led her to similar conclusions regarding the use of a modular grid in the Taj Mahal (Koch 2006: 108-109) and the length of the Mughal yard (Koch 1991 and 2006).

This use should be understood in the context of a category of mathematical and geometric methods from pre-Euclidean times for the expression of root powers ($\sqrt{2}$, $\sqrt{3}$, or $\sqrt{5}$) as a succession of whole-number ratios. “These successive ratios approach nearer and nearer to the root

Table 1

Divisions of $\sqrt{2}$ & a	$a\sqrt{2}$	a	$\frac{a\sqrt{2}}{2}$	$\frac{a}{2}$	$\frac{a\sqrt{2}}{4}$	$\frac{a}{4}$	$\frac{a\sqrt{2}}{8}$	$\frac{a}{8}$	$\frac{a\sqrt{2}}{16}$	$\frac{a}{16}$	$\frac{a\sqrt{2}}{32}$	$\frac{a}{32}$
Modular Approx.	10	7	5	3.5	2.5	1.75	1.25	7/8	5/8	7/16	5/16	7/32

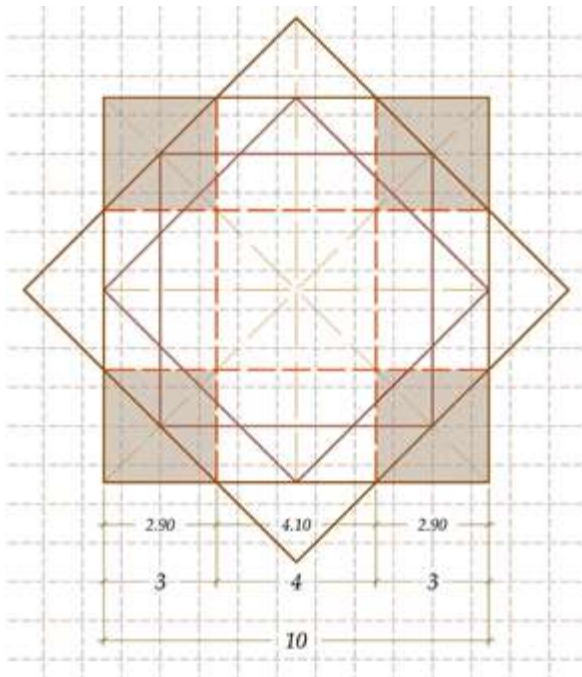


Figure 29: *Hasht Bihisht* division of a 7x7 square

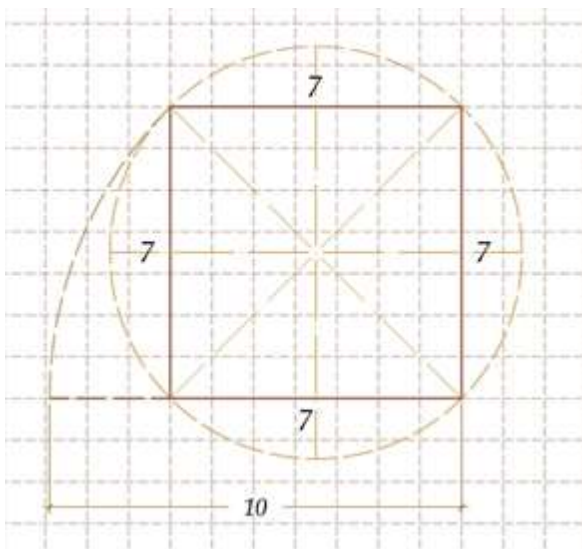


Figure 30: Approximation of the diagonal of a 7x7 square

value with each alternation” (Lawlor 1994: 39). Likewise, number series were used to generate approximations of root functions, the Fibonacci series being a well-known example, for generating the golden ratio. “Similarly a method known as the Diaphantine method—the generation of whole number ratios approximating incommensurable functions—is attributed to the Greek mathematician Diaphantus” (Lawlor, 1994: 67). The works of Diaphantus were translated into Arabic by Qusta ibn Luqa in the tenth century (Nasr 1968: 149).

In the case of Bulatov’s analyses, the irrational geometry is approximated in terms of the module and its subdivisions.

Table 1 (where a = generative unit) shows a set of such conversions, given by Bulatov (1988: 112) and borne out by his analyses of Central Asian monuments (Fig. 28).

According to Golombek and Wilber (1988: 140), the geometric system used in Timurid architecture “... aside from its practical value as a working method, ensured a harmony of parts, whereby all parts were related to a single entity.”

A grid of square “units” naturally lends itself to graphically representing these mathematical methods. As an illustration of the use of the grid to approximate incommensurable measurements resulting from root functions (the diagonal of the square in this example), figure 29 shows the *Hasht Bihisht* division of a square of 7x7 units. This is given by the intersections of two rotating squares as well as by the arc formed from the square’s corners to the midpoint of its diagonal to the side of the square. The diagonal of a square of 1x1 unit is $\sqrt{2}$ units, i.e., the irrational number 1.414213..., hence the half-diagonal $\sqrt{2}/2$ for a square of 7x7 units would be $\sqrt{2}/2 \times 7 = 4.949747$.

The smaller portion of the octagonal division (hatched in Fig. 29) is $7 - 4.949747 = 2.05025$ (rounded to 2.05 in Fig. 29), while the central division is $7 - (2 \times 2.05025 \dots) = 2.89949$ (rounded to 2.90 in Fig. 29). The grid gives us whole-number approximations of these, i.e., 2, 3, and 2 (Fig. 29).

Figure 30 shows the approximation of the diagonal of a square of 7x7 units. The diagonal of a square of 1x1 unit is $\sqrt{2}$ units, i.e., the irrational number 1.414213..., hence the diagonal for a square of 7x7 units would be $\sqrt{2} \times 7 = 9.8994949 \dots$ In figure 29 the grid gives the approximation of the diagonal as 10.

Bulatov’s conclusions regarding the connection between mathematics, architecture, and geometry, as well as the above grid method, are understandable when we recall that historically there was a close connection between architecture and geometry. “According to the tenth century philosopher, Abu Nasr al-Farabi, the fundamentals of architecture belonged to the mathematical sciences” (Golombek and Wilber 1988: 137). Golombek and Wilber also state that “Geometry was the foundation of an architect’s training, and the highly skilled architect was known as *muhandis*, a “geometer.”

That Mughal architects were similarly skilled is confirmed by the information we have about them. We know this regarding Ustad Ahmad Mimar Lahori, *chief architect during the reign of Shah Jahan*, and from Begley and Desai’s *Anthology of Seventeenth-Century Mughal and European Documentary Sources* (1989), on the Taj Mahal and its designers. The family of Ustad Ahmad were not only architects but also geometers, mathematicians, and astronomers, as well as poets. This is attested to by the

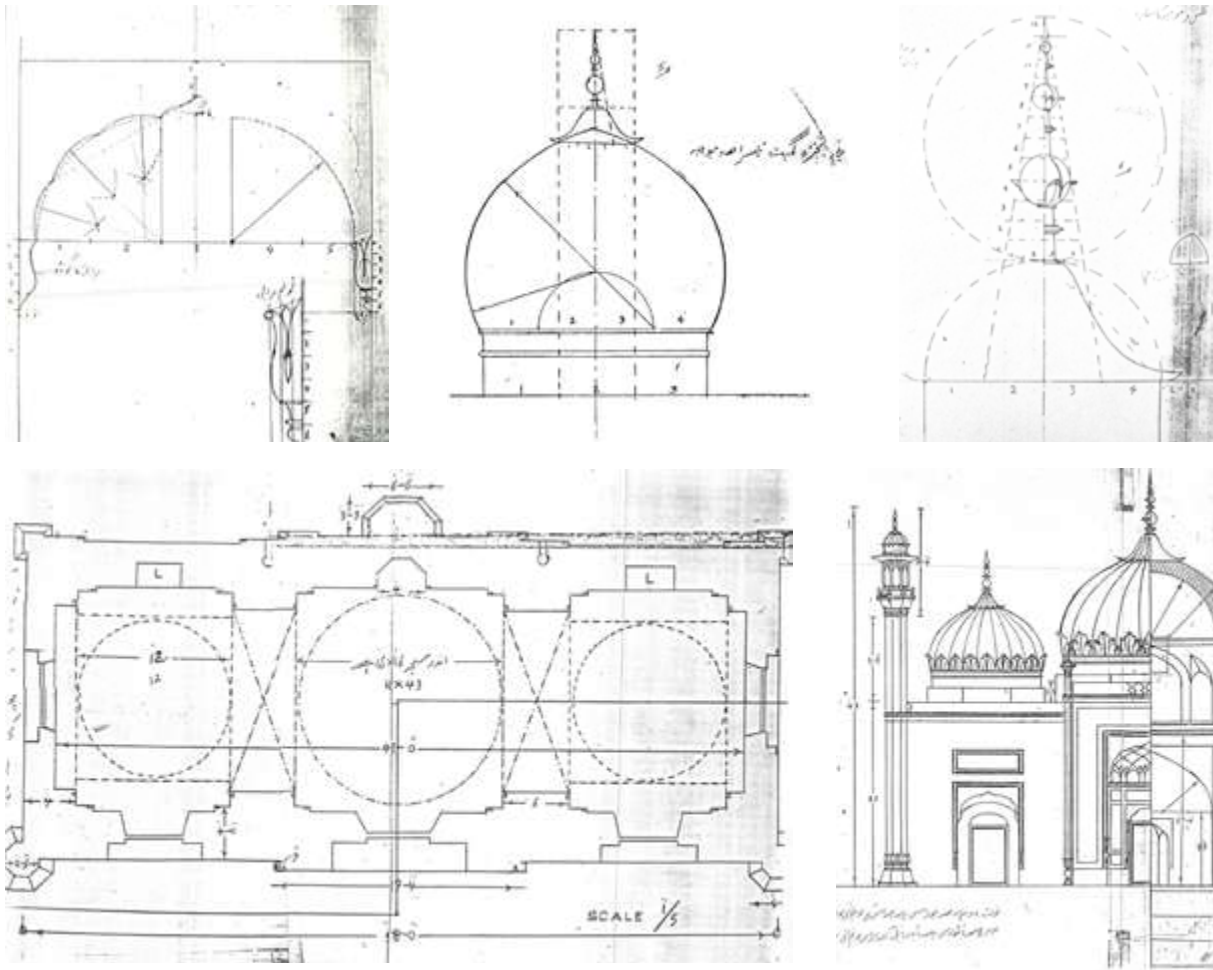


Figure 31: Merabi (arch) in “Punjabi” shape (Ustad 1970s)

Figure 32: 4-part dome with 1/3 part *maujbah* (inverted lotus) (Ustad 1970s)

Figure 33: *Kalas* (finial) and *maujbah* (inverted lotus) proportioning system (Ustad 1970s)

Figure 34: Multan mosque ground plan (Ustad 1977)

Figure 35: Multan mosque half sectional elevation (Ustad 1977)

following verses from Ustad Ahmad’s youngest son Lutf Allah’s collection *Diwan-i Muhandis*, “The Geometer’s (*Muhandis*) Poetry Collection (*Diwan*)” (Begley and Desai 1989):

3. Ahmad the architect (*mimar*), who in his art
Was a hundred steps ahead of the masters of this art.

4. Conversant with Euclid’s Propositions (*Tahrir*) and
its Discourses,
And acquainted with all their particulars.

5. The movements of the planets and stars had become
known to him,
The mysteries of the *Almagest* had been understood by
him.

9. At the orders of the world-conquering king,
He constructed the edifice of the tomb of Mumtaz Mahal
(*Taj Mahal*).

14. From the wonderful man, three sons remained,
Of those three, *Ata Allah Rushdi* (or *Rashidi*) is the
eldest.

15. He was a cherisher of skill and the master of his
profession
Accomplished and learned, and a savant of the Age.

23. I am the second of those three brothers;
Mathematics (*hindsa*) is only one of my hundred arts.

25. The third of those three brothers, in years,
Is *Nur Allah*, the Master of Perfection.

26. We all are architects and builders;
We all are master cherishers of speech (*poets*).

31. The treasure of art has come within his grasp;
He is an expert calligrapher in all the seven scripts (*haft-qalam*).

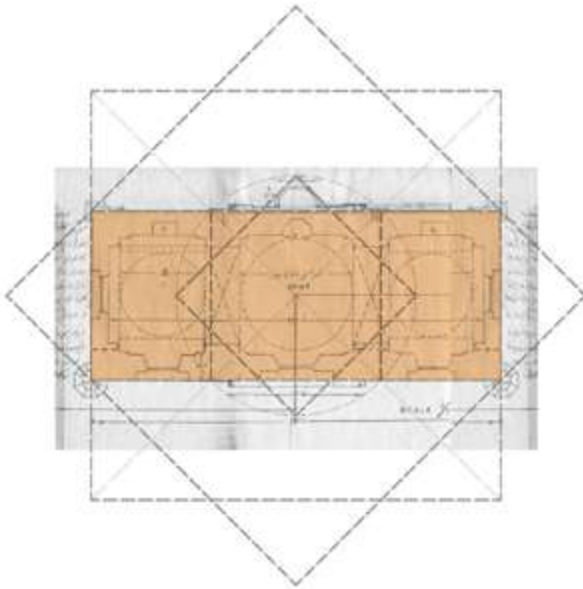


Figure 36: Geometric analysis by Ustad (1977) of a mosque ground plan

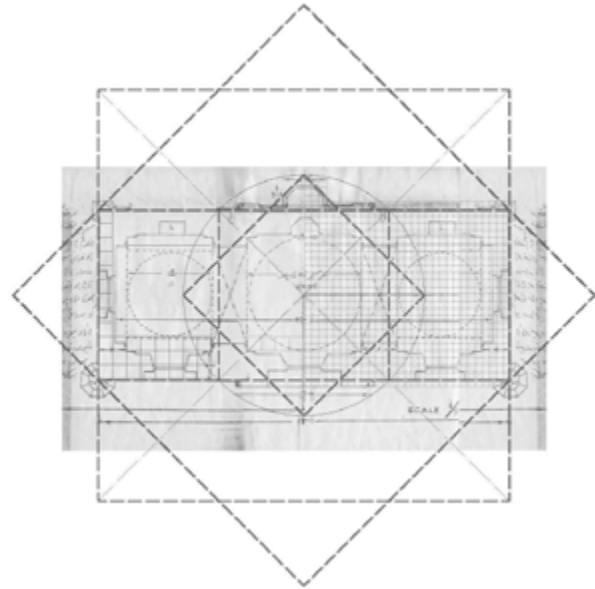


Figure 37: Grid analysis by Ustad (1977) of a mosque ground plan

*33. Even though Muhandis [geometrician] is my title,
You should instead seek geometry from those two
brothers.*

One also finds evidence of the above manner of working in an unpublished twentieth-century 30-page handbook of assorted drawings (Figs. 31-33) by the late master mason Ustad Rahim Bukhsh, of the historic Pakistani city of Multan. The drawings of various elements of traditional mosque architecture clearly show the impact of geometry on the design of such elements, both as a practical tool for drawing or constructing forms such as domes, arches, geometric patterns etc., and as a means of creating a harmony of parts through proportioning.

Two of the drawings have dates on them: 1974 and 1977, probably the last. The final set is of a complete ensemble, namely a mosque (Figs. 31 and 32), making a logical conclusion, as all the elements illustrated—arches, domes, *minars*, *chatris*, etc.—thereby come together to make a whole.

Analysis of the Mosque Ground Plan

The ratio between the mosque's length and width—58 ft : 24 ft, i.e., 17.68 m : 7.32 m—accurately expresses the irrational relationship $A : A(\sqrt{2} - 1)$ geometrically determined by the intersections of two squares rotated at 45 degrees (Fig. 36). A grid of $2'' \times 2''$ overlaid on the left quarter of the mosque plan and another of $1'' \times 1''$ on its right half clearly show that the planning follows a network based on imperial feet and inches (standard units used in contemporary building practice) (Fig. 37).

The analyses presented here show the continuity of these principles and methods not only between the classical Mughal period and the work of contemporary master masons but also further back in Central Asian architecture, common perhaps with broader Islamic design (Tabba 1988). This continuity appears in Koch's considerations (1990) regarding Mughal architecture's "Timurid link". In addition, in the sixteenth century, Central Asia was seen as the touchstone of cultural standards and taste, and as a result the early Mughal buildings were openly modelled on Timurid ideals (Koch 1990). Lastly, we may recall that the Muslim community in the region—Central Asia, Transoxiana, Iran, Afghanistan, and India—was a cultural continuum (Richard C. Foltz 1998).

Our findings point to the possibility of taking this research further by applying these methods to other buildings so as to get a more nuanced understanding of design processes used in the Subcontinent. This is in addition to the usefulness of such analyses in learning to use these methods and principles in contemporary design.

Conclusion: Interpretation in the Light of Traditional Cosmology

Bulatov (1978 and 1988) as well as Golombek and Wilber (1988) in the case of Timurid architecture, and Koch in the case of the Mughal tradition (2006), have pointed to the need to place architecture in the cultural context of Islamic civilization and its ideals of beauty and harmony. We too have tried to do this. Taking this logic further, it would be fruitful to situate these design methods in the framework of traditional Islamic cosmology, as this cosmology springs

from the same worldview in which this architecture was created and which indeed underpins all the various traditional Islamic arts and sciences (Nasr 1978).

Islamic cosmology, like other premodern cosmologies, sees creation as having two main aspects: on the one hand, “quantitative,” “material,” and “measurable,” and on the other, “qualitative,” “immeasurable,” and “essential.” In the Islamic tradition this duality is often represented by the terms *surah* and *ma’na*—“outer form” and “essence-reality.”

This is the world of “form” (surat) and we live in “forms”;
The “Essence-Reality” (ma’na) cannot be seen save by means of “form.”

Awhad al-Din Kirmani (1163–1238)

This is analogous to other traditions. For example, in Hindu culture, *Purusha* and *Prakriti* (Stoddart 1993) stand for these principles in relation to the whole of creation, and

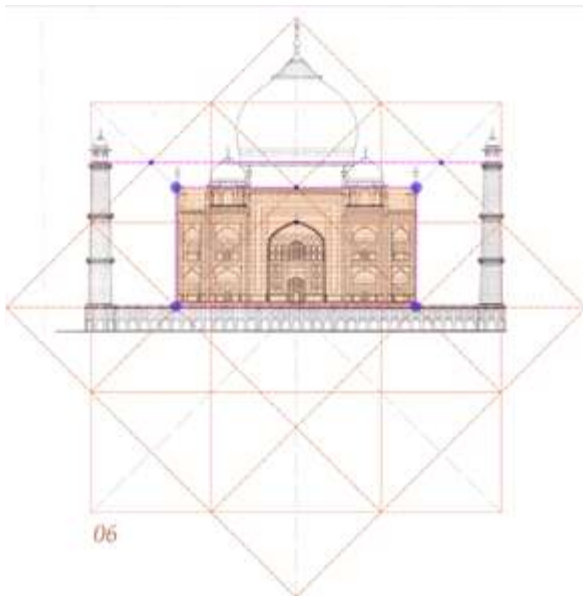


Figure 38: Taj Mahal (partial analysis). The basic *Hasht Bihisht* framework of two rotating squares at 45 degrees and the lines joining their intersections give us many of the monument’s main widths and heights (Koch 2006: 154)

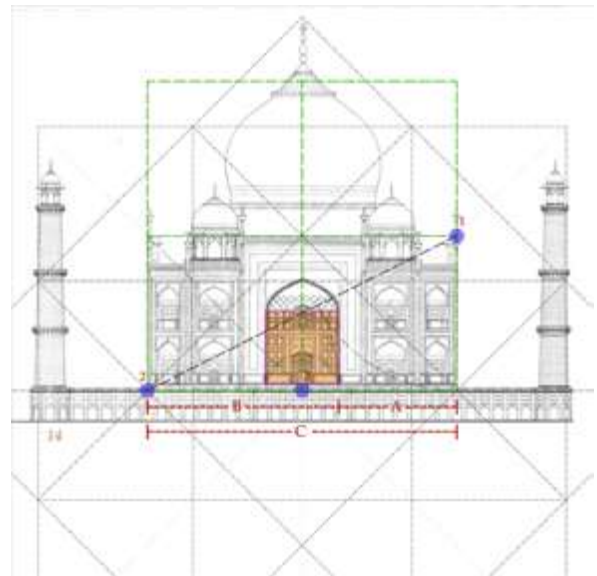


Figure 39: Taj Mahal (partial analysis): use of the “golden cut” for establishing the central *ivan* (double-height archway) width. Using the Euclidean method of bisecting the semi-diagonal of a double-square rectangle, the length C is divided into lengths A and B, where the ratio of A:B : B:C is equal to the golden ratio (Koch 2006: 154)

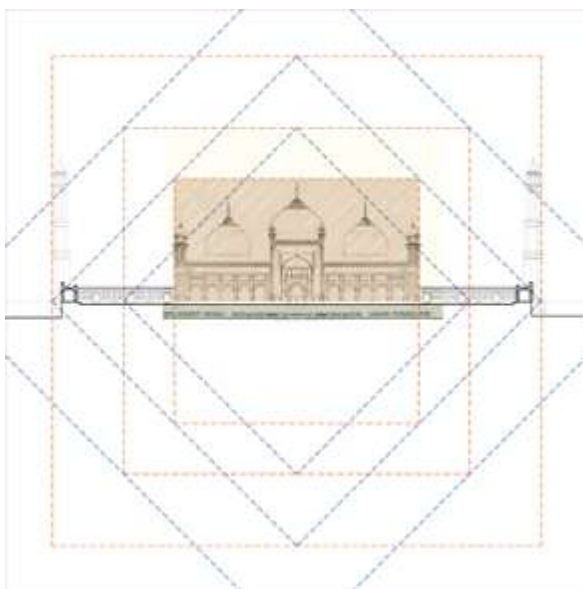


Figure 40: Badshahi Mosque (partial analysis): starting from the edges of the outer minarets, a series of inscribed rotating squares brings us to the mosque’s width and height in three steps (Chughtai 1957)

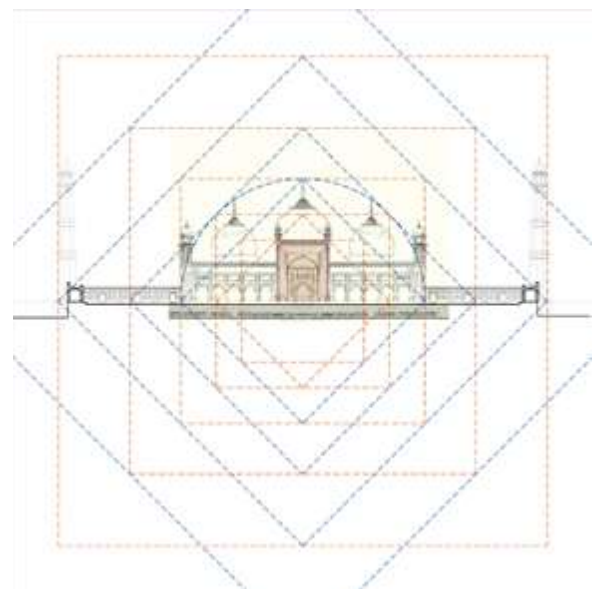


Figure 41: Badshahi Mosque (partial analysis): continuing the series of inscribed rotating squares further inward gives us the width and height of the central *ivan* (double-height central archway) (Chughtai 1957)

naam (“name,” “essential reality”) and *rupa* (“outer form”) in relation to individual entities (Coomaraswamy 1934). In the Aristotelian hylomorphic cosmological paradigm, adopted by the Islamic tradition, *hyle* (“wood,” meaning “matter/material”) and *morphe* (“form,” meaning “essence” or “quiddity,” i.e., “what” a thing is, e.g., “man,” “tree,” etc.), stand for these two aspects (Smith 1995). The medieval Western scholastic terminology for them—*materia et forma*—is based on the Aristotelian conception received through Muslim philosophers such as Averroes (Ibn Rushd) and Avicenna (Ibn Sina) (Stanford Encyclopedia of Philosophy 2022).

Consider a marble statue. The marble is the matter of the statue whereas the shape signifies the “form” of the statue. The marble is the “stuff” out of which the statue is made whereas the shape signifies the “form” that the artist decided to give to the statue. On a more metaphysical level, “form” is the principle whereby “matter” has the particular structure that it has, and “matter” is simply that which stands to be structured in a certain way. (Internet Encyclopedia of Philosophy).

The equivalent terms for *materia* and *forma* suggested by the French Muslim Sufi philosopher René Guénon in *The Reign of Quantity and the Signs of the Times* (1945) are “substance” for *materia* and “essence” for *forma*, and as applied to materially existing entities, he suggests the terms “quantity” and “quality.” This framework is paralleled in the concepts and methods of traditional Islamic art and architecture (Burckhardt 1976), as in the arts of other traditions (Coomaraswamy 1934).

Applying this framework to Mughal architecture would mean that the use of the grid refers to the “measurable,” “quantitative” aspect, while geometric proportioning refers to the “qualitative” aspect, the source and “essence” of beauty and harmony.

According to this cosmological scheme, we could say that the study of the various planning grids at the Taj Mahal discerned by Ebba Koch (2006: 108-109) shows primarily the “quantitative” half of design. The planning grids were “measuring” and “laying-out” tools. The “qualitative” half, the source of timeless beauty, lies in its underlying geometric proportioning. We believe this to be demonstrated by our analysis of the two tombs above, and of the two elevations presented below.

In conclusion, we may recall that the “universal” cosmology outlined above was overturned on the adoption of Descartes’s scientific method (Smith 1995), inaugurating the scientific revolution but also leading to the Cartesian split in Western philosophy’s conception of reality. The world and with it humankind were divided into irreconcilable compartments, namely “quantity” and “quality,” with the difference that only the “quantitative” aspect was considered objectively “real,” because of being

“measurable,” while “qualities” (including beauty) were relegated to the “subjective” realm. This scientific method, which underpins modern academic enquiry, including in art history, arguably limits us to one half of reality. For a meaningful art-historical investigation doing justice to its subject and bringing full appreciation, we should perhaps look at art and architecture in accordance with the worldview from which they emerge.

In our analysis of two Mughal tombs we looked at two plans and one section, and so we now end with analytical elevations of two outstanding Mughal monuments: the Taj Mahal (1632-1648) in Agra (Figs. 38 and 39)⁵, built by Shah Jahan, and the Badshahi Mosque (1671-1673) in Lahore (Figs. 40 and 41), built in the reign of Shah Jahan’s successor Aurangzeb Alamgir, once again combining the measurable and the immeasurable.

¹ Timur, born in 1336 in Kesh near Samarkand (now in Uzbekistan), is chiefly remembered for his conquests from India and Russia to the Mediterranean and his dynasty’s cultural achievements.

² Geometric tools are used both as practical methods for drawing or constructing forms such as domes, arches, or geometric patterns and as instruments for creating harmony of parts through proportioning.

³ *Gaz* is the Mughal linear yard (also called *zira*). The prevailing *gaz* for architecture was the *gaz-i ilahi*, introduced under Akbar. In Shah Jahan’s time its length was 80-82 cm (Koch 2006: 260).

⁴ The *gaz* yard has been shown to be the *gaz-illahi* introduced under Akbar (81-82 cm), with remarkable consistency between the reigns of Akbar and Shah Jahan.

⁵ The full scheme would be the subject of another paper.

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